

Written Examination
 Graduate School of Marine Science and Technology
 Course of Maritime Technology and Logistics
 Basic Subject Maritime Technology and Logistics
 February 3, 2026

Purpose of the Questions

This examination evaluates the basic mathematical skills necessary for graduate-level research. The following examples are limited to answers that can be written as numbers, formulas, or short sentences. Equivalent correct expressions are treated equally. In addition to the correctness of the answers, the intermediate steps are also taken into account for the evaluation.

- I Evaluate the student's knowledge of eigenvalues and eigenvectors of matrices and ability to compute them.
- II Evaluate the student's knowledge of extremum problems and ability to compute them.
- III Evaluate the student's knowledge of integrals and ability to compute them.
- IV Evaluate the student's knowledge of ordinary differential equations and ability to compute them.

Solutions

I

- (1) (20 points) The eigenvalues of A are -1 , 2 , and the corresponding eigenvectors are

$$c_1 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + c_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad ((c_1, c_2) \neq (0, 0)), \quad c_3 \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix} \quad (c_3 \neq 0), \text{ respectively.}$$

- (2) (5 points) For example, $P = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$.

II (25 points) By solving $f_x(x, y) = f_y(x, y) = 0$, we obtain stationary points $(x, y) = (1, -1)$, $\left(\frac{5}{12}, \frac{1}{6}\right)$. By considering the signs of $D(x, y) = f_{xy}(x, y)^2 - f_{xx}(x, y)f_{yy}(x, y)$, $f_{xx}(x, y)$ at these points, f has a local minimum -1 at $(x, y) = (1, -1)$, and f does not have extrema at $(x, y) = \left(\frac{5}{12}, \frac{1}{6}\right)$.

III

- (1) (12 points) $\log|x - 1| + \text{Arctan}(x + 1) + C$, where C is an arbitrary constant.
- (2) (13 points) $\frac{3}{2} \log 3 - 2 \log 2$

IV

(1) (12 points) $y = C \exp\left(-(1-x^2)^{\frac{3}{2}}\right)$, where C is an arbitrary constant.

(2) (13 points) $y = \left(\frac{x}{2} + \frac{9}{4}\right) e^{-x} + C_1 e^x + C_2 e^{-\frac{2}{3}x}$, where C_1, C_2 are arbitrary constants.